TRINITY COLLEGE

Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

SOLUTIONS

Section One: Calculator-free

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(7 marks)**

(a) The distance run A_n km by an athlete in the n^{th} week of training for an event is given by the recurrence relation $A_{n+1} = A_n + 5.5,\ A_1 = 16$.

(i) Use the relation to complete the following table. (2 marks)

n	1	2	3	4	5
A_n	**16**	**21.5**	**27**	**32.5**	**38**

Solution
See table
Specific behaviours
✓ at least two correct entries
✓ all correct entries

(ii) What name is given to such a sequence of terms that model linear growth? (1 mark)

Solution
Arithmetic sequence.
Specific behaviours
✓ correct sequence name

(b) The distances run by another athlete in each of the first four weeks that they trained for the event form the sequence $15, 21, 27$ and 33 km.

(i) Deduce a rule for the n^{th} term of this sequence of distances. (2 marks)

Solution
$T_n = 15 + 6(n-1) = 6n + 9$
Specific behaviours
✓ indicates correct common difference
✓ correct rule (any n^{th} term form)

(ii) In which week of training will this athlete first run at least 80 km? Justify your answer. (2 marks)

Solution
$6n + 9 = 80$ $6n = 71$ $n = 11\frac{5}{6}$ During the 12^{th} week.
Specific behaviours
✓ correct week
✓ justifies by continuing sequence, forming equation, etc

See next page

Question 2 **(8 marks)**

Car owners who experienced a theft related to their car were asked which state they lived in and whether insurance covered the theft. The responses of 800 owners are shown below.

	Did insurance cover your theft?			
State	Yes	No, claim rejected	No, not insured	Total
Victoria	144	176	**80**	400
South Australia	**25**	100	75	**200**
Western Australia	110	**50**	40	200

(a) Name the two categorical variables in the table. (1 mark)

Solution
'State' and 'Did insurance cover your theft'.
Specific behaviours
✓ names both variables

(b) Complete the two-way table above. (2 marks)

Solution (b)
See table
Specific behaviours
✓ two correct values
✓ all correct values

(c) Use the theft data to complete the following row percentaged table. (3 marks)

	Did insurance cover your theft?		
State	Yes	No, claim rejected	No, not insured
Victoria	**36**	**44**	**20**
South Australia	**12.5**	**50**	**37.5**
Western Australia	**55**	**25**	**20**

Solution
See table
Specific behaviours
✓ one correct row
✓ all rows have total of 100
✓ all rows correct

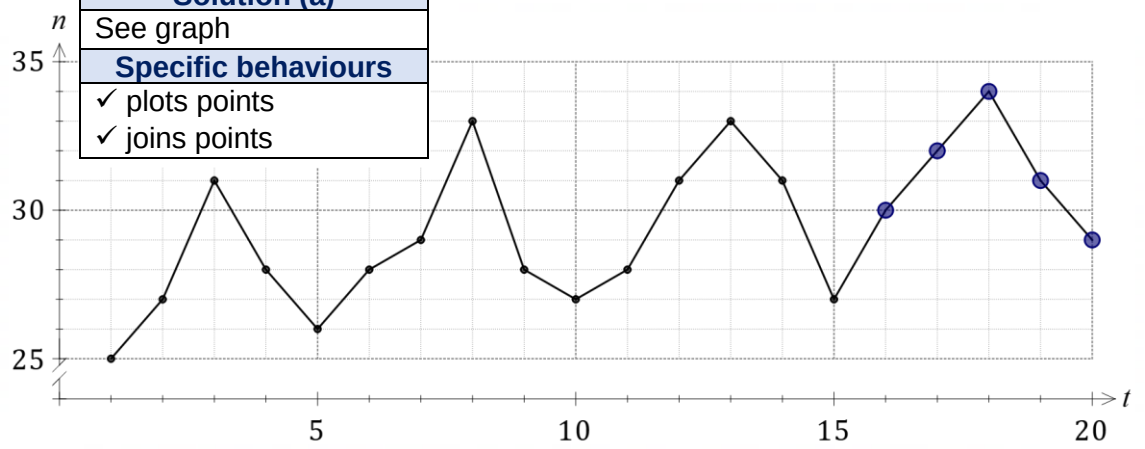
(d) Does the data suggest the presence of an association between the two variables? Justify your answer. (2 marks)

Solution
Yes. 55% of those who live in WA were covered by insurance compared to only 30% in SA and 36% in Victoria.
Specific behaviours
✓ states yes
✓ justifies by illustrating different percentages in any column

See next page

Question 3 **(7 marks)**

An office is open five days a week (from Wednesday to Sunday inclusive), and the plot below shows the number of clients n visiting the office on each of 15 consecutive business days.



Solution (a)
See graph
Specific behaviours
✓ plots points
✓ joins points

During the following week, the data below was recorded:

Day	Wed	Thu	Fri	Sat	Sun
Time, t	16	17	18	19	20
Number of clients, n	30	32	34	31	29

(a) Use the data for the fourth week to complete the time series plot above. (2 marks)

(b) Describe the trend and seasonality of the time series. (2 marks)

Solution
The trend of the time series is increasing. Seasonality: Clients increase from Wednesday to a high on Friday, and then drop to a low every Sunday.
Specific behaviours
✓ states trend is increasing
✓ refers to high and low seasons

Using the average percentage method, the seasonal indices for Wednesday, Thursday, Friday and Sunday were calculated as $0.94, 1.01, 1.12$ and 0.93 respectively. The equation of the trend line for the number of clients visiting the office is $y = 0.2t + 26.9$.

(c) Determine the seasonal index for Saturday. (1 mark)

Solution
$0.94 + 1.01 + 1.12 + 0.93 = 1.95 + 2.05 = 4$ Index for Saturday is $5 - 4 = 1$.
Specific behaviours
✓ correct index

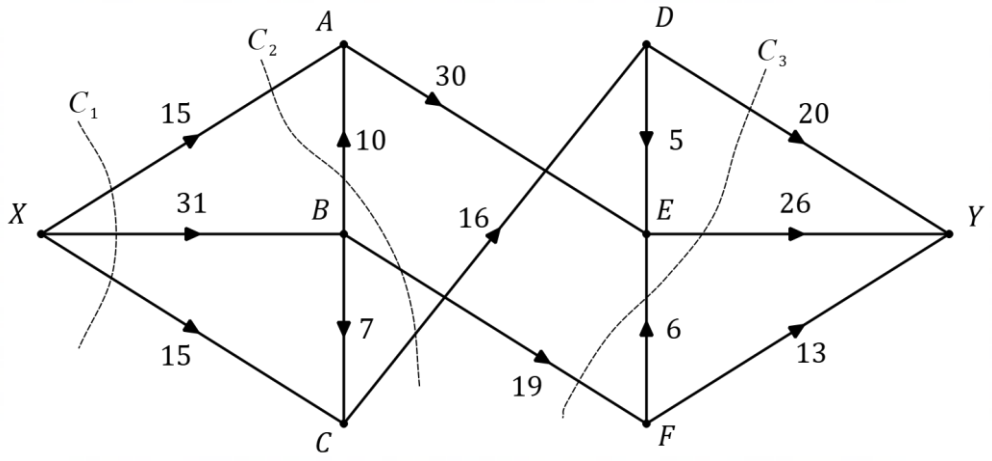
(d) Write down a calculation that will best predict the number of clients expected to visit the office when $t = 23$. Do **not** evaluate your answer. (2 marks)

Solution
$(0.2(23) + 26.9) \times 1.12$
Specific behaviours
✓ indicates correct seasonal index
✓ correct calculation with appropriate use of brackets

See next page

Question 4 **(6 marks)**

The directed network below shows a system of conveyor belts used to move minerals from a receiving depot at X to a ship at Y. The edge weights indicate the tonnes of minerals that each conveyor belt can move per hour and the three dotted lines indicate cuts.



(a) Determine the capacity of each of the three cuts shown above. (2 marks)

Solution
$C_1 = 15 + 31 + 15 = 61,$ $C_2 = 15 + 10 + 19 + 16 = 60,$ $C_3 = 20 + 26 + 0 + 19 = 65.$
Specific behaviours
✓ one correct cut capacity ✓ all correct capacities

(b) Determine the maximum weight of minerals that can be moved from the receiving depot to the ship every hour. (3 marks)

Solution
$XAEY = 15$ $XBAEY = 10$ $XBFEY = 1$ $XBFY = 13$ $XBCDY = 7$ $XCDY = 9$ Maximum weight is $15 + 10 + 1 + 13 + 7 + 9 = 55$ tonnes per hour. (Edges with spare capacity: $XC = 6, AE, BF, DE, FE = 5, DY = 4$)
Specific behaviours
✓ indicates systematic approach using cuts or flows ✓ shows all cuts or all flow paths ✓ correct maximum

(c) State the maximum possible increase to your answer in part (b) if the capacity of the conveyor belt from D to E could be increased by up to 4 tonnes per hour. (1 mark)

Solution
No increase is possible as DE already has spare capacity.
Specific behaviours
✓ indicates no increase possible

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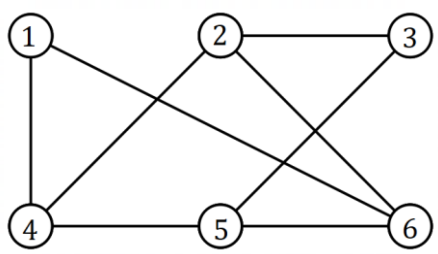
Question 5 **(8 marks)**

(a) In the bipartite graph shown below, one set of vertices represents swimmers, and the other set represents the events that they will compete in. If vertex 1 is an event, determine which of the vertices represent a swimmer. (2 marks)



Solution
Each edge joins a swimmer to an event. Vertices $3, 4$ and 6 represent swimmers.
Specific behaviours
✓ indicates correct approach ✓ correct set of vertices

(b) A connected planar graph has 15 vertices and 21 edges. Determine the number of faces the graph has. (2 marks)

Solution
$v + f - e = 2$ $15 + f - 21 = 2$ $f = 2 + 21 - 15 = 8$ Graph has 8 faces.
Specific behaviours
✓ indicates use of Euler's formula ✓ correct number of faces

(c) If possible, draw a simple connected graph that has 3 vertices and 4 edges. If not possible, explain why not. (2 marks)

Solution
Not possible – a complete graph with 3 vertices will only have 3 edges. The fourth edge will have to be a loop or multiple edge and so graph will not be simple.
Specific behaviours
✓ states not possible ✓ reasonable explanation

(d) There are $\frac{n}{2}(n-1)$ edges in a complete graph with n vertices. Determine the number of edges that must be added to a tree with 10 vertices so that it becomes a complete graph. (2 marks)

Solution
Tree will have $10 - 1 = 9$ edges. K_{10} has $\frac{1}{2}(10)(9) = 45$ edges. Must add $45 - 9 = 36$ edges to become complete.
Specific behaviours
✓ indicates correct number of edges for tree or K_{10} ✓ correct number of edges to add

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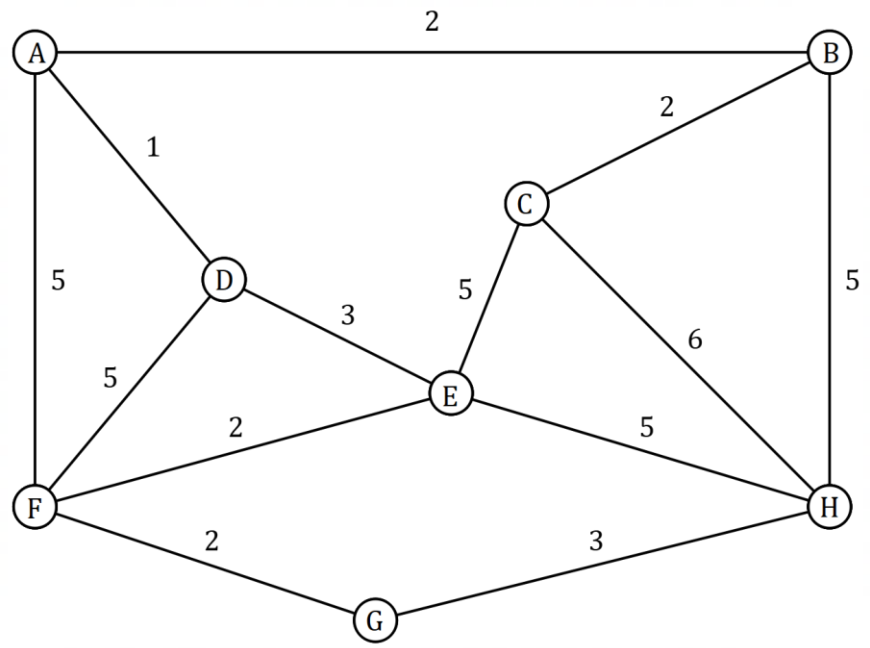
Question 6 **(9 marks)**

In the graph shown below, the vertices represent the eight branches of a bank in a city, the edges indicate that a courier service exists between adjacent branches and the weight on each edge is the number of sets of traffic lights that a courier travelling from one branch to another must pass through.



(a) Determine the number of zeros in the adjacency matrix for the graph. (2 marks)

Solution
There are 13 edges, no multiple edges or loops, and so there will be $2 \times 13 = 26$ ones. There are 8 vertices, and so the matrix will contain $8 \times 8 = 64$ entries, and so $64 - 26 = 38$ zeros.
Specific behaviours
✓ indicates total number of entries in matrix ✓ correct number of zeros

(b) Explain why the graph cannot be Eulerian. (1 mark)

Solution
Not all of the vertices have an even degree.
Specific behaviours
✓ correct explanation

(c) Explain why the graph is Hamiltonian. (2 marks)

Solution
The graph contains a cycle (closed path) through all the vertices.
Specific behaviours
✓ contains cycle ✓ through all vertices

See next page

(d) A courier must collect a document from branch G, visit each of the other seven branches exactly once to get it signed and then return the document to branch G.

(i) Determine two possible orders in which the courier could visit the branches.

(2 marks)

Solution
There are six possible orders, those below and their reverses: G – F – **A – D – E – C – B** – H – G G – F – **D – A – B – C – E** – H – G G – F – **E – D – A – B – C** – H – G
Specific behaviours
✓ one correct order ✓ second correct order (possibly reverse of first)

(ii) Determine the least number of sets of traffic lights that the courier could pass through and state the corresponding order in which the branches are visited.

(2 marks)

Solution
The route G – F – E – D – A – B – C – H – G (or reverse) passes though the least number of sets of lights. $2 + 2 + 3 + 1 + 2 + 2 + 6 + 3 = 21$ sets of lights. *Route in (d) starting with GFA is* 26 *and GFD is* 25*.*
Specific behaviours
✓ correct minimum number ✓ correct order

See next page

Question 7 **(7 marks)**

One Sunday evening, a catering company has to assign four food trucks, 1, 2, 3 and 4, to four locations, A, B, C and D. Each truck must be assigned to exactly one location for the entire evening. Truck 3 cannot be assigned to location D and truck 4 cannot be assigned to location C.

The table below shows the profit, in hundreds of dollars, that each truck is expected to make at each location.

	A	**B**	**C**	**D**
1	36	41	40	37
2	43	42	44	40
3	41	42	44	–
4	43	40	–	41

(a) Use the Hungarian algorithm to determine an assignment of trucks to locations that will maximise the total profit made by the four trucks. (6 marks)

Solution

Response formatted as series of tables, grids, matrices, etc:

$$\begin{bmatrix}36 & 41 & 40 & 37\\ 43 & 42 & 44 & 40\\ 41 & 42 & 44 & 0\\ 43 & 40 & 0 & 41\end{bmatrix} \rightarrow \begin{bmatrix}8 & 3 & 4 & 7\\ 1 & 2 & 0 & 4\\ 3 & 2 & 0 & 44\\ 1 & 4 & 44 & 3\end{bmatrix}$$

$$\begin{bmatrix}5 & 0 & 1 & 4\\ 1 & 2 & 0 & 4\\ 3 & 2 & 0 & 44\\ 0 & 3 & 43 & 2\end{bmatrix} \rightarrow \begin{bmatrix}5 & 0 & 1 & 2\\ 1 & 2 & 0 & 2\\ 3 & 2 & 0 & 42\\ 0 & 3 & 43 & 0\end{bmatrix} \rightarrow \begin{bmatrix}4 & \mathbf{0} & 1 & 1\\ \mathbf{0} & 2 & 0 & 1\\ 2 & 2 & \mathbf{0} & 41\\ 0 & 4 & 44 & \mathbf{0}\end{bmatrix}$$

Assignment of trucks to locations is $1 \rightarrow B, 2 \rightarrow A, 3 \rightarrow C, 4 \rightarrow D$.

Specific behaviours

✓ replaces both dashed entries with number ≤ 36
✓ subtracts all entries from 44
✓ reduces rows and columns
✓ covers zeros with three lines
✓ applies algorithm using least uncovered number of 1
✓ states correct assignment of trucks to locations

(b) State the resulting total profit. (1 mark)

Solution

$43 + 41 + 44 + 41 = 169$.
Hence total profit is $\$16\,900$.

Specific behaviours

✓ correct total profit

End of questions

Supplementary page

Question number: _____